Mathematical modeling and the physics of electric field on non-symmetric linear charge density for damped-oscillatory motion

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บทคัดย่อ

งานวิจัยนี้ แสดงถึงการคำนวณหาสนามไฟฟ้าบนประจุเชิงเส้นที่มีความหนาแน่นประจุเป็นแบบไม่สมมาตร และมีการสั่นแบบหน่วง โดยใช้วิธีการอินทิเกรตทีละส่วน จนกระทั่งได้ผลลัพธ์เป็นสนามไฟฟ้าที่เป็นฟังก์ชันของระยะทาง รวมทั้งได้แสดงกราฟของสนามไฟฟ้า และระยะทางที่ขึ้นอยู่กับค่าคงที่ λ_0

คำสำคัญ: การอินทิเกรตทีละส่วน, ประจุเชิงเส้นที่มีความหนาแน่นประจุเป็นแบบไม่สมมาตร

Abstract

In this research, non-symmetric linear charge density with damped oscillatory motion was studied in order to solve the electric field using integration by parts technique. The numerical results for electric field as a function of length were obtained. The results were plotted and compared with the varying damping coefficient and λ_0 constant

Keyword: integration by parts, non-symmetric linear charge density

Introduction

The gravitational field $\vec{g}$ at a point in space was defined in Newton's law of universal gravitation to be equal to the gravitational force $\vec{F}_g$ acting on a test particle of mass m_0 divided by the mass of the test particle : $\vec{g} = \vec{F}_g / m_0$. This is the gravitational version of the model of a particle in a field. In similar manner, an electric field at a point in

space can be defined in terms of the electric force acting on a test particle with charge q_0 placed at that point. Because charge exists in two varieties, therefore we choose the convention that a test particle always carries a positive electric charge. With this convention, The researcher can introduce the electric version of the particle in a field model—the electric field E at a point in space is defined as the electric force F_e acting on a test particle placed at that point divided by the charge q_0 of the test particle : $E \times q_0 \equiv F_e$ (D.J. Griffiths 1999). The work by A. Maglaras and L. Maglaras in 2014 was to evaluate the electric field distribution in the model of rod plate air gap with a charged barrier under different geometries and arrangements with finite element method. F. Zuo 2014, had shown a geometrical to evaluate the electric field due to a uniformly charged rod by integration method. E.R. Generazio's syudy in 2017 was to construct the photograph of positive and negative electrode of the rotating quasi-static dipole element and measurements electric field imaging may be applied to characterize intrinsic electric potential and field. Thus, an electric field exists at a point if a charged test particle placed at rest at that point experiences an electric force. In this work, we study the numerical results of electric field due to a charged rod with non-symmetric linear charge density $(\lambda(x) = \lambda_0 \frac{x^3}{l^2} e^{-i\beta x} \cos(\pi x))$ where λ_0 is a constant and β is the damping coefficient.

The electric field due to a charged rod with non-symmetric linear charge density as a function of the position

A rod of length ℓ has a uniform linear charge density λ ($\lambda(x) = \lambda_0 \frac{x^3}{\ell^2} e^{-i\beta x} \cos(\pi x)$)(P.Lorrain and D. R. Corson 1962) and a total charge Q . Calculate the electric field at a point P along the axis of the rod, a distance a from one end.

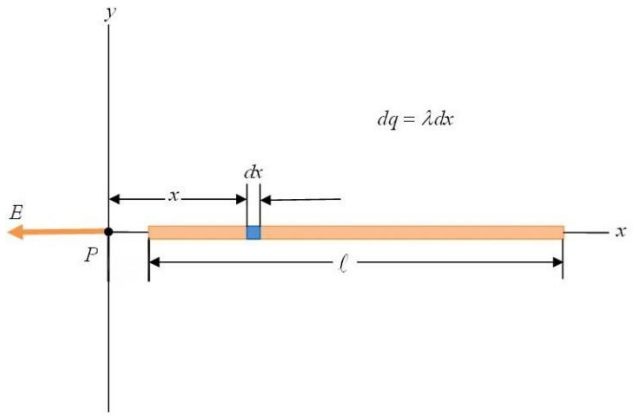


Figure 1The electric field at P due to a uniformly charged rod lying along the x axis. The field at P due to the segment of charge dq is $k\,dq/x^2$. The total field at P is the vector sum over all segment of the rod.

Next, we express the charge dq of the element in terms of the other variables within the integral. If necessary, the integral is expressed in terms of components. Then we reduce the integral to one over a single variable. For this calculation, the rod is taken to be along the $\boldsymbol{x}$ axis. Let us use dx to represent the length of one small segment of the rod and let dq be the charge on the segment. The charge dq on the small segment is $dq = \lambda_0 \frac{x^3}{\ell^2} e^{-i\beta x} \cos(\pi x)\, dx.$ The field $d\mathbf{E}$ (C. A. Brau 2004) due to this segment at the point P is in the negative $\boldsymbol{x}$ direction, and magnitude is

$$dE = \frac{-kdq}{R^2} = -k\frac{\lambda_0 \frac{x^3}{\ell^2} e^{-i\beta x} \cos(\pi x)\, dx}{x^2} \qquad (1)$$

Each element of the charge distribution produces a field in the negative x direction, and so the vector sum of their contributions reduces to an algebraic sum. The total field at P due to all segment of the rod, which are at different distances from P, is given by Equation , which in this case becomes

$$E = \frac{-k\lambda_0}{\ell^2} \cdot \int xe^{-i\beta x} \cos(\pi x)\, dx \tag{2}$$

Where the limits on the integral extend from one end of the rod ($x = 0$) to the other ($x = \ell$). Because k and λ are constants, they can be removed from the integral. Thus, the researcher find this is first equation, The researcher have

$$E = \frac{-k\lambda_0}{\ell^2} \cdot \int_0^{\ell} xe^{-i\beta x} \cos(\pi x)\, dx \tag{3}$$

Choose one of the three factors and set it equal to u. Then put equal to the remaining factor time dv . For definiteness, suppose that we have chosen to put $u = x$, so that $dv = e^{-i\beta x} \cos(\pi x)\, dx$. Calculate $du = dx$ and calculate $v = \int e^{-i\beta x} \cos(\pi x)\, dx$. Do not bother to write a constant of integration for the latter integral. The researcher now have $u = x, \quad du = dx$ and $dv = e^{-i\beta x} \cos(\pi x)\, dx$

$$v = \int e^{-i\beta x} \cos(\pi x)\, dx = \frac{\pi}{(\pi^2 - \beta^2)}\left[e^{-i\beta x} \sin(\pi x)\right] - \frac{i\beta}{(\pi^2 - \beta^2)}\left[e^{-i\beta x} \cos(\pi x)\right]$$

The final total electric field for linear charge density asymmetric solution then is

$$E = \frac{-\hat{i}\, k\lambda_0}{\ell^2} \cdot \left(\left(\frac{e^{-i\beta\ell}}{(\pi^2 - \beta^2)^2}\left(\left((\pi^2 - \beta^2)\ell\pi + 2i\beta\pi\left[\sin(\pi\ell)\right]\right) - \left((\pi^2 - \beta^2)\ell i\beta - \left(\pi^2 + \beta^2\right)\left[\cos(\pi\ell)\right]\right)\right)\right) - \left(\frac{\pi^2 + \beta^2}{(\pi^2 - \beta^2)^2}\right)\right)$$

(4)

The researcher obtained numerical the electric field E in rod (length) is section Numerical and results

Numerical results

The researcher find that the solution of the equation of motion and of the electric field due to a charged rod of linear charge density with damping are the length function. Such the electric field due to a charged which depend on the damping coefficient (β) and the parameter λ_0 constant for oscillatory motion are shown in Figure (2)- (3).

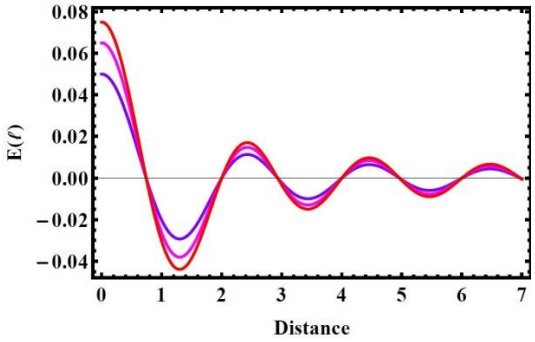


Figure 2 Representation of the electric field due to a charged rod for real number oscillatory motion of electron with vary λ_0 and setting the parameter $\beta = 0.05,\ k = 1$. Purple is $\beta = 0.05$, $\lambda_0 = 0.1$, pink is $\beta = 0.05$, $\lambda_0 = 0.13$, red is $\beta = 0.05$, $\lambda_0 = 0.15$

In figure 2, when we increase λ_0 from 0.1 to 0.13 and 0.13 to 0.15. It will influence the electric field due to a charged rod to be escalated.

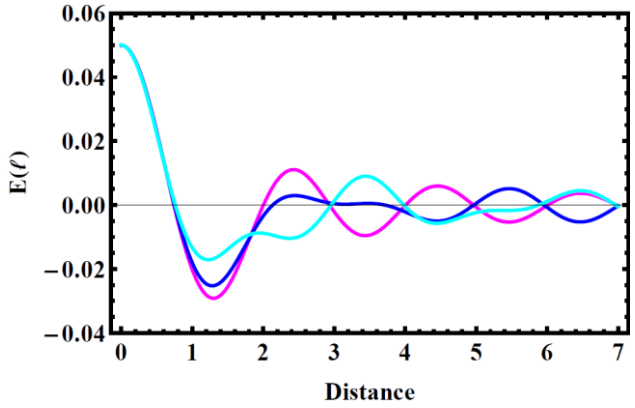


Figure 3 The electric field due to a charged rod for real number oscillatory motion of electron as a function of distance with vary β and $\beta = 0.1, 0.5, 0.9$, $\lambda_0 = 0.1$, $k = 1$, pink is $\beta = 0.1$, $\lambda_0 = 0.1,$ blue is $\beta = 0.5$, $\lambda_0 = 0.1,$ light blue is $\beta = 0.9$, $\lambda_0 = 0.1$

When the damping coefficient β was varied from 0.1 to 0.5 and 0.5 to 0.9 the electric field was found to decrease accordingly as shown in figure 3.

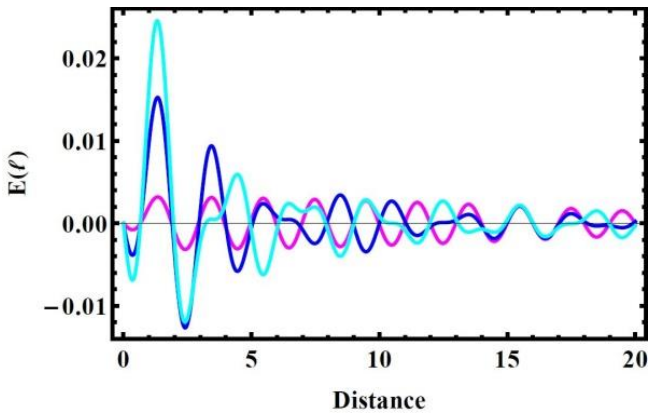


Figure 4 The electric field due to a charged rod for image number oscillatory motion of electron as a function of distance with vary β and $\beta = 0.1, 0.5, 0.9$, $\lambda_0 = 0.1$, $k = 1$ pink is $\beta = 0.1$, $\lambda_0 = 0.1,$ blue is $\beta = 0.5$, $\lambda_0 = 0.1,$ light blue is $\beta = 0.9$, $\lambda_0 = 0.1$

When we supplement coefficient β from0.1 to 0.5 and 0.5 to 0.9. It will influence the electric field due to a charged rod to be declined as shown in figure 4.

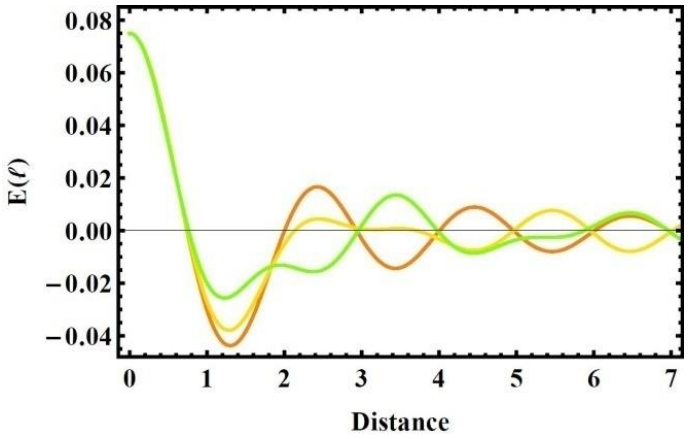


Figure 5 The electric field due to a charged rod for real number oscillatory motion of electron as a function of distance with vary β $\beta = 0.1, 0.5, 0.9$, $\lambda_0 = 0.15$, $k = 1$, orange is $\beta = 0.1$, $\lambda_0 = 0.15$, yellow is $\beta = 0.5$, $\lambda_0 = 0.15$, green is $\beta = 0.9$, $\lambda_0 = 0.15$

Relate to figure 5. when we increase damped coefficient (β) from0.1 to 0.5 and 0.5 to 0.9. It will influence the electric field due to a charged rod to be declined.

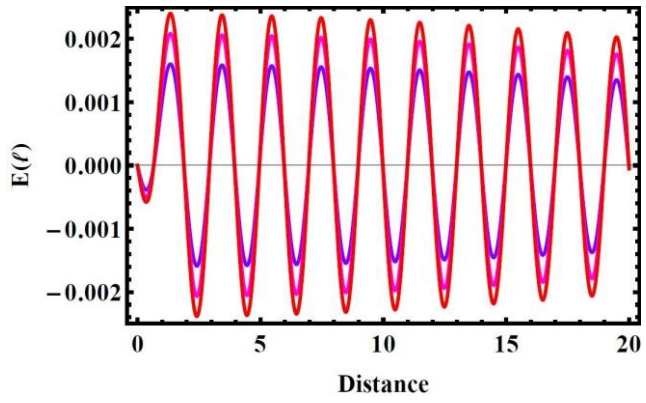


Figure 6 The electric field due to a charged rod for image number oscillatory motion of electron as a function of distance with vary λ_0

Relate to figure 6.when we supplement λ_0 from 0.1 to 0.13 and 0.13 to 0.15. It will influence the electric field due to a charged rod to be escalated.

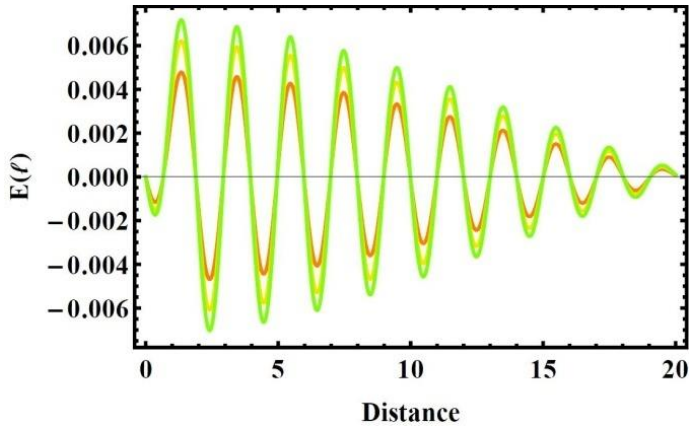


Figure 7 The electric field due to a charged rod for image number oscillatory motion of electron as a function of distance with vary λ_0 $\beta = 0.15$, $\lambda_0 = 0.1, 0.13, 0.15$, $k = 1$, orange is $\beta = 0.15$, $\lambda_0 = 0.1$, yellow is $\beta = 0.15$, $\lambda_0 = 0.13$, green is $\beta = 0.15$, $\lambda_0 = 0.15$

Relate to figure 7, when we supplement from 0.1 to 0.13 and 0.13 to 0.15. It will influence the electric field due to a charged rod to be escalated.

Conclusion

Summary, from figure 2, figure 3, figure 4, when the length pass electric field's vibration that are length's function will depend by function $\cos(\pi\ell)$ and $\sin(\pi\ell)$ but electric field's amplitude will diminish on factor $e^{-i\beta\ell}$

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